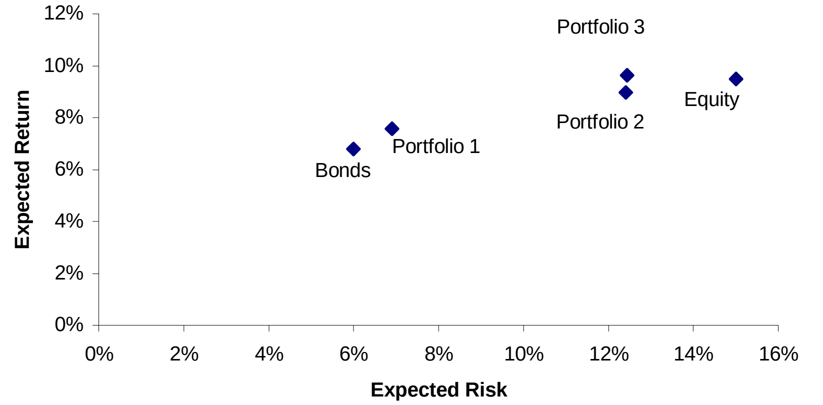
IS THE SYSTEMATIC USE OF LEVERAGED ETFS THE MYTHICAL "FREE LUNCH"?

The traditional approach to asset allocation has been to change the relative proportion of risky assets to arrive at a desired level of risk. One would increase the weight of risky asset classes at the expense of less volatile ones to increase a portfolio's desired risk return and vice versa to arrive at a more conservative portfolio. A person with a low risk budget and conservative preferences would traditionally select something like a 20% equity/80% bond portfolio, and a moderate investor – a 60%/40% mix. Somebody with an even higher appetite for risk would increase exposure to emerging markets and other high-risk high-return asset classes. On the extreme side, plunging into emerging markets might result in a sufficient level of risk. The flipside, however, are concentrated portfolios with high idiosyncratic risk, fat tails, and negative skewness. In addition, foregoing the diversification benefits of fixed income, the risk-seeking client gives up the "free lunch" that diversification provides. In other words shifting a portfolio into higher gear by simply increasing the percentage invested into higher risk asset classes reduces the Sharpe ratio and results in a suboptimal portfolio.

Expected Return and Risk for Various Asset Classes and Portfolios



But the problem still persists: there are clients with under-spent risk budgets who would like to assume more risk in an optimal manner. The solution is to add very moderate amounts of leverage, now available without the hassle and expense of opening margin accounts, through the use of innovative products such as BetaPro bull funds. This increases the risk/return of the portfolio to sufficient levels without reducing its Sharpe ratio, increasing kurtosis and subjecting a portfolio to idiosyncrasies of higher risk asset classes.

	Return	Risk	WEIGHTS Portfolio 1	Portfolio 2	Portfolio 3
Equities	9.5%	15%	29%	80%	51%
Bonds	6.8%	6%	71%	20%	129%
Leverage			0%	0%	80%
Return			7.6%	9.0%	9.6%
Risk			6.9%	12.4%	12.4%
Sharpe			0.37	0.32	0.37
VAR			-8.5%	-20.0%	-19.4%

EXAMPLE : Here is a very simple example that demonstrates how leverage can improve return without increasing risk. Assume an economy where all asset classes have a constant Sharpe ratio of 0.3. With a risk free rate of 5%, correlation between equity and bonds of 30% and standard deviations of 15% and 6% respectively, we calculate the expected returns of 9.5% and 6.8% that correspond to the above Sharpe ratio.

Now let's find an optimal portfolio with the highest return per unit of risk. Under our assumptions, this portfolio would be invested in 30% equities and 70% bonds. This results in a Sharpe ratio of 0.37, which is higher than either of the two asset classes due to the less than perfect correlation. This portfolio is suitable for a client with an annual risk budget (annual VAR at 1% confidence level) of 8.5%. Let's say, however, that the client's risk budget is higher, at 20%. If we don't use leverage, the optimal portfolio that consumes this risk budget efficiently consists of 80% equity and 20% bonds with an expected return of 9% and

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standard deviation of 12.4%. The Sharpe ratio of this portfolio, however, is lower at only 0.32 since we are not taking full advantage of low correlation that exists between equity and bonds.

The other approach would be to leverage the first portfolio 80%. Let's see the effect that this has on the target portfolio. Firstly we observe that the Sharpe ratio remained the same at 0.37 since leverage preserves the Sharpe ratio. Secondly, we can see that risk is the same as the one without leverage, while returns are 60 bps higher! Free lunch? No, leverage simply takes better advantage of diversification benefits. Lower correlation between equity and bonds allows risk to be reduced, leaving room in the risk budget. This room is further used to enhance return. The result is a portfolio with the same level of risk and higher return.

This example is based on a simplistic case of only two asset classes. Increasing the number of asset classes allows investors to reap further benefits of diversification and improves returns even more. The expanding array of ETFs available to individual investors creates new opportunities.

Is leverage only for investors with a high risk budget? Not necessarily. Moderate amounts of leverage can benefit conservative investors as well, improving their diversification and creating more efficient portfolios. Investors refusing to use leverage limit their opportunity set and fail to reap the full benefits of diversification.

Leverage is traditionally associated with higher risk. And while generally true, this is not always the case. “Dollars invested” is not a good measure of risk! That’s why leverage by itself does not equate with risk. In fact, it can mean **lower** risk if used appropriately. As investors try to increase expected return, the universe of asset classes from which they can choose diminishes and portfolios become less and less diversified. Notice that the amount of leverage employed by this approach is very moderate, with a maximum 20%-30%. In fact, moderate amounts of leverage allow investors to create a highly diversified portfolio, which are less risky than unlevered undiversified ones.

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November 2007

APPENDIX

Denote expected return of an asset class as *r* and it’s volatility as σ . Denote risk free rate as r_f . Then the Sharpe ratio for this asset class is :

$$SR = \frac{r - r_f}{\sigma}$$

Denote leverage factor as *L*. For example, *L*=1.2 means that the investment is levered by 20% or, equivalently, exposure is equal to 120%.

The return of levered investment is :

$$r(L) = r_f + L(r - r_f)$$

and its volatility is :

$$\sigma(L) = L\sigma$$

Then the Sharpe ratio of a levered asset class is :

$$SR(L) = \frac{r_f + L(r - r_f) - r_f}{L\sigma} = \frac{r - r_f}{\sigma} \equiv SR$$

The fact that leverage does not change the Sharpe ratio of an asset class certainly also holds true for a portfolio as a whole. Thus an optimal portfolio can be levered or de-levered to create any desired amount of risk while still taking full advantage of diversification benefits offered by available asset classes

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